Global stability analysis in delayed cellular neural networks

Jinde Cao
Adult Education College, Yunnan University, Kunming 650091, People's Republic of China

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In this paper, the author analyzes further problems of global stability for a class of cellular neural networks with delays by means of the Lyapunov functional method, inequalities $a^2+b^2\geqslant 2ab$ and $a^3+b^3+c^3\geqslant 3abc$ $(a,b,c\geqslant 0)$ analysis technique, some stability criteria are obtained under more general conditions. These criteria can be applied to design globally stable networks and thus have important significance in both theory and applications. [S1063-651X(99)04605-X]

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I. INTRODUCTION

Cellular neural networks (CNN's) were introduced by Chua and Yang [1,2] in 1988. They found important applications in signal processing, especially in image treatment. It is well known that the CNN is formed by many units called cells, and the structure of the CNN is similar to that found in cellular automata, namely, any cell in a cellular neural network is connected only to its neighbor cells. A cell contains linear and nonlinear circuit elements, which typically are linear capacitors, linear resistors, linear and nonlinear controlled sources, and independent sources. For the circuit diagram and implementing the connection pattern for the CNN the reader is referred to [1]. The CNN can be applied in signal processing, moreover it can be used to solve some image processing and pattern recognition problems, but it is necessary to solve some moving image processing and pattern recognition by using cellular neural networks with delay (DCNN). The DCNN can only be described by delayed differential equations (namely, functional differential equations), in fact, the CNN is described by ordinary differential equations. The studies of stability of the DCNN and CNN are of theoretical and applicable significance in the design of networks. There exist some results of stability for CNN and DCNN, and we refer to [1–7] and the references cited therein. The purpose of this paper is to derive some more general sufficient conditions for the global asymptotic stability of the DCNN, which are independent of delays, by using the Lyapunov functional method [7–9], inequalities $a^2+b^2\geqslant 2ab$ and $a^3+b^3+c^3\geqslant 3abc$ $(a,b,c\geqslant 0)$ analysis technique. Thus, some results related in references [1–7], and the references cited therein, are extended and improved. These are of theoretical and applicable significance in signal processing, especially in moving image treatment and the design of networks.

In the following, we consider the DCNN model described by differential equations with delays

$$x_i'(t)=-c_ix_i(t)+\sum_{j=1}^{n} a_{ij}f_j(x_j(t))+\sum_{j=1}^{n} b_{ij}f_j(x_j(t-\tau_j))+I_i,$$

$$c_i>0,\quad i=1,2,\ldots,n, \tag{1}$$

in which n corresponds to the number of units in a neural network; $x_i(t)$ corresponds to the state vector of the ith unit at time t; $f_j(x_j(t))$ denotes the output of the jth unit at time t; a_{ij}, b_{ij}, I_i, c_i are constant, a_{ij} denotes the strength of the jth unit on the ith unit at time t, b_{ij} denotes the strength of the jth unit on the ith unit at time $t-\tau_j$, I_i denotes the external bias on the ith unit, τ_j corresponds to the transmission delay along the axon of the jth unit and is not a negative constant, c_i represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs. In the following, we assume that each of the relations between the output of the cell f_i $(i=1,2,\ldots,n)$ and the state of the cell possess the following properties:

(H_1) f_i $(i=1,2,\ldots,n)$ is bounded on R;

(H_2) There is a number $\mu_i>0$ such that $|f_i(u)-f_i(v)|\leqslant\mu_i|u-v|$ for any $u,v\in R$.

It is easy to find from (H_2) that f_i is a continuous function on R. In particular, if the relation between the output of the cell and the state of the cell is described by a piecewise-linear function $f_i(x)=\frac{1}{2}(|x+1|-|x-1|)$, then it is easy to see that the function f_i clearly satisfies the hypotheses (H_1) and (H_2) above, and $\mu_i\equiv 1$ $(i=1,2,\ldots,n)$. The circuit implementing of Eqs. (1) can be referred to Refs. [1,3].

II. STABILITY ANALYSIS ON THE DCNN

Lemma 1. Assume that the output of the cell function f_i $(i=1,2,\ldots,n)$ satisfies the hypotheses (H_1) and (H_2) above. Then there exists an equilibrium for the DCNN (1).

Proof. If $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T$ denotes an equilibrium of the DCNN(1), then x^* satisfies the nonlinear algebraic system

$$x_i^*(t)=c_i^{-1}\left[\sum_{j=1}^{n} a_{ij}f_j(x_j^*)+\sum_{j=1}^{n} b_{ij}f_j(x_j^*)+I_i\right]=\sum_{j=1}^{n}[c_i^{-1}(a_{ij}+b_{ij})]f_j(x_j^*)+c_i^{-1}I_i. \tag{2}$$

Let $B=[c_i^{-1}(a_{ij}+b_{ij})]_{n\times n}$, $I=(c_1^{-1}I_1, c_2^{-1}I_2, \ldots,$

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$c_n^{-1}I_n)^T$, $f(x^*)=[f_1(x_1^*),\ f_2(x_2^*),\dots,f_n(x_n^*)]^T$. Then the system (2) can be written in vector-matrix notation in the form

$$x^*=F(x^*)=Bf(x^*)+I, \tag{3}$$

thus x^* is a fixed point of the map $F:R^n\to R^n$. The existence of a fixed point of the map F can be shown by the well-known Brouwer's fixed-point theorem. In fact, the ith component of $F(x)$ satisfies the following form:

$$|(F(x))_i|=\left|\sum_{j=1}^n [c_i^{-1}(a_{ij}+b_{ij})]f_j(x_j)+c_i^{-1}I_i\right|$$

$$\leqslant\sum_{j=1}^n |c_i^{-1}(a_{ij}+b_{ij})||f_j(x_j)|+c_i^{-1}|I_i|$$

$$\leqslant\sum_{j=1}^n |c_i^{-1}(a_{ij}+b_{ij})|M+c_i^{-1}|I_i|,$$

where $x=(x_1,x_2,\dots,x_n)$, $M=\max_{1\leqslant i\leqslant n}\sup_s|f_i(s)|$. Let $K=\max_{1\leqslant i\leqslant n}(\Sigma_{j=1}^n|c_i^{-1}(a_{ij}+b_{ij})|M+c_i^{-1}|I_i|)$, then $F(R^n)\subset Q=\{(x_1,x_2,\dots,x_n)\in R^n||x_i|\leqslant K,\ i=1,2,\dots,n\}$. It is easy to see that the map F is continuous. Let the map F be restricted on Q, namely $F|_Q:Q\to Q$. Obviously $F|_Q$ maps the bounded closed and convex set Q on R^n into itself; hence by the well-known Brouwer's theorem, the map F has at least one fixed point say $x^*=(x_1^*,x_2^*,\dots,x_n^*)^T$. This completes the proof.

Remark 1. We note that Brouwer's theorem does not guarantee the uniqueness of the fixed point. However, in this paper, we derive some sufficient criteria on the system (1) which will guarantee not only the uniqueness of the equilibrium but also its recall by its global asymptotic stability. The uniqueness of the equilibrium will follow from the global asymptotic stability to be established below.

Lemma 2. For the DCNN (1), suppose that the output of the cell f_i $(i=1,2,\dots,n)$ satisfies the hypotheses (H_1) and (H_2) above. Then all solutions of the DCNN (1) remain bounded for $[0,+\infty)$.

Proof. It is easy to observe that all solutions of the DCNN (1) satisfy differential inequalities of the form

$$-c_ix_i(t)-\delta_i\leqslant x_i'(t)\leqslant -c_ix_i(t)+\delta_i, \tag{4}$$

where $\delta_i=\Sigma_{j=1}^n(|a_{ij}|+|b_{ij}|)\sup_{s\in R}|f_j(s)|+|I_i|$.

Using Eq. (4) above, one can easily prove that solutions of the DCNN (1) remain bounded on $[0,+\infty)$. This completes the proof.

Theorem 1. For the DCNN (1), suppose that the outputs of the cell f_i $(i=1,2,\dots,n)$ satisfy the hypotheses (H_1) and (H_2) above. Assume, furthermore, that the system parameters a_{ij},b_{ij} $(i,j=1,2,\dots,n)$ satisfy the conditions

$$\sum_{j=1}^n (|a_{ij}|\mu_j^{2\alpha_j^*}+|a_{ji}|\mu_i^{2\beta_i^*}+|b_{ij}|\mu_j^{2\alpha_j}+|b_{ji}|\mu_i^{2\beta_i})<2c_i,$$

in which $\alpha_j^*,\beta_j^*,\alpha_j,\beta_j$ $(j=1,2,\dots,n)$ are any real numbers with $\alpha_j^*+\beta_j^*=1$, $\alpha_j+\beta_j=1$, and μ_j $(j=1,2,\dots,n)$ being the constant numbers of the hypotheses (H_2) above.

Then the equilibrium x^* of the DCNN (1) is globally asymptotically stable independent of delays.

Proof. If $x^*=(x_1^*,x_2^*,\dots,x_n^*)^T$ is an equilibrium of the DCNN (1), one can derive from Eq. (1) that the deviations $y_i(t)=x_i(t)-x_i^*$ $(i=1,2,\dots,n)$ satisfy

$$y_i'(t)=-c_iy_i(t)+\sum_{j=1}^n a_{ij}[f_j(x_j^*+y_j(t))-f_j(x_j^*)]$$
$$+\sum_{j=1}^n b_{ij}[f_j(x_j^*+y_j(t-\tau_j))-f_j(x_j^*)]; \tag{5}$$

obviously, $(0,0,\dots,0)^T$ is an equilibrium of Eq. (5). To prove the global asymptotic stability of x^* of the DCNN (1), it is sufficient to prove the global asymptotic stability of the trivial solution of Eq. (5). The local existence of solutions of Eqs. (1) and (5) follows by the method of steps, the existence of $[0,+\infty)$ will be a consequence of our analysis below. Now we consider the Lyapunov functional defined by

$$V(t)=V(y)(t)$$
$$=\sum_{i=1}^n\left(\frac{1}{2}y_i^2(t)+\frac{1}{2}\sum_{j=1}^n |b_{ij}|\mu_j^{2\beta_j}\int_{t-\tau_j}^t y_j^2(s)ds\right), \tag{6}$$

where β_j $(j=1,2,\dots,n)$ are any real numbers.

Calculating the derivative of V along the solution of Eq. (5), we get

$$\frac{dV}{dt}=\sum_{i=1}^{n}\Bigg[y_i(t)-c_iy_i(t)+\sum_{j=1}^{n}a_{ij}[f_j(x_j^*+y_j(t))-f_j(x_j^*)]+\sum_{j=1}^{n}b_{ij}[f_j(x_j^*+y_j(t-\tau_j))-f_j(x_j^*)]$$

$$+\frac{1}{2}\sum_{j=1}^{n}|b_{ij}|\mu_j^{2\beta_j}[y_j^2(t)-y_j^2(t-\tau_j)]\Bigg]$$

$$\leqslant\sum_{i=1}^{n}\Bigg[-c_iy_i^2(t)+\sum_{j=1}^{n}|a_{ij}|\mu_j|y_i(t)||y_j(t)|$$

$$+\sum_{j=1}^{n}|b_{ij}|\mu_j|y_i(t)||y_j(t-\tau_j)|+\frac{1}{2}\sum_{j=1}^{n}|b_{ij}|\mu_j^{2\beta_j}[y_j^2(t)-y_j^2(t-\tau_j)]\Bigg]$$

$$=\sum_{i=1}^{n}\Bigg[-c_iy_i^2(t)+\sum_{j=1}^{n}|a_{ij}|(\mu_j^{\alpha_j^*}|y_i(t)|)(\mu_j^{\beta_j^*}|y_j(t)|)+\sum_{j=1}^{n}|b_{ij}|(\mu_j^{\alpha_j}|y_i(t)|)(\mu_j^{\beta_j}|y_j(t-\tau_j)|)$$

$$+\frac{1}{2}\sum_{j=1}^{n}|b_{ij}|\mu_j^{2\beta_j}[y_j^2(t)-y_j^2(t-\tau_j)]\Bigg]. \tag{7}$$

$\alpha_j^*,\beta_j^*,\alpha_j,\beta_j\ \ (j=1,2,\ldots,n)$ are any real numbers with $\alpha_j^*+\beta_j^*=1,\ \ \alpha_j+\beta_j=1$. Now estimating the right side of Eq. (7) by using the inequality $2ab\leqslant a^2+b^2$, we have

$$\frac{dV}{dt}\leqslant\sum_{i=1}^{n}\Bigg[-c_iy_i^2(t)+\sum_{j=1}^{n}|a_{ij}|\frac{1}{2}[(\mu_j^{\alpha_j^*}|y_i(t)|)^2+(\mu_j^{\beta_j^*}|y_j(t)|)^2]$$

$$+\sum_{j=1}^{n}|b_{ij}|\frac{1}{2}[(\mu_j^{\alpha_j}|y_i(t)|)^2+(\mu_j^{\beta_j}|y_j(t-\tau_j)|)^2]+\frac{1}{2}\sum_{j=1}^{n}|b_{ij}|\mu_j^{2\beta_j}[y_j^2(t)-y_j^2(t-\tau_j)]\Bigg]$$

$$=\sum_{i=1}^{n}\Bigg[-c_i+\frac{1}{2}\sum_{j=1}^{n}(|a_{ij}|\mu_j^{2\alpha_j^*}+|a_{ji}|\mu_i^{2\beta_i^*}+|b_{ij}|\mu_j^{2\alpha_j}+|b_{ji}|\mu_i^{2\beta_i})\Bigg]y_i^2(t)$$

$$\leqslant-r\sum_{i=1}^{n}y_i^2(t), \tag{8}$$

where $r=\min_{1\leqslant i\leqslant n}[-(c_i-\frac{1}{2}\Sigma_{j=1}^{n}(|a_{ij}|\mu_j^{2\alpha_j^*}+|a_{ji}|\mu_i^{2\beta_i^*}+|b_{ij}|\mu_j^{2\alpha_j}+|b_{ji}|\mu_i^{2\beta_i}))]>0$.

A consequence of Eq. (8) is that

$$V(y)(t)+r\int_0^t\sum_{i=1}^{n}y_i^2(s)ds\leqslant V(y)(0). \tag{9}$$

It follows from Eq. (9) that

$$\int_0^{+\infty}\sum_{i=1}^{n}y_i^2(t)dt<+\infty. \tag{10}$$

According to Lemma 2, $x_i(t)$ is bounded on $(0,+\infty)$. This implies boundedness of $y_i(t),y_i'(t)$ on $(0,+\infty)$; hence $y_i(t),y_i^2(t)$ are uniformly continuous on $(0,+\infty)$. By Barbalat's Lemma [10], it follows that

$$\lim_{t\to+\infty}\sum_{i=1}^{n}y_i^2(t)=0. \tag{11}$$

It follows from Eq. (11) that the zero solution of Eq. (5) is globally asymptotically stable for any delays, thus the equilibrium of the DCNN (1) is also globally asymptotically stable. This completes the proof.

Theorem 2. For the DCNN (1), suppose that the output of the cell $f_i\ \ (i=1,2,\ldots,n)$ satisfies the hypotheses (H_1) and (H_2) above. Assume furthermore that the system parameters $a_{ij},b_{ij}\ \ (i,j=1,2,\ldots,n)$ satisfy the following conditions:

$$\sum_{j=1}^{n}[(|a_{ij}|\mu_j^{3\alpha_j}+|a_{ij}|\mu_j^{3\beta_j}+|a_{ji}|\mu_i^{3\gamma_i})$$

$$+(|b_{ij}|\mu_j^{3\alpha_j}+|b_{ij}|\mu_j^{3\beta_j}+|b_{ji}|\mu_i^{3\gamma_i})]<3c_i,$$

in which $\mu_j\ \ (j=1,2,\ldots,n)$ are the constant numbers of the hypotheses (H_2) above, and $\alpha_j, \beta_j, \gamma_j$ are any real numbers with $\alpha_j+\beta_j+\gamma_j=1$ $(j=1,2,\dots,n)$.

Then the equilibrium x^* of the DCNN (1) is also globally asymptotically stable independent of delays.

Proof. Consider the Lyapunov functional defined by

$$V(t)=V(y)(t)=\sum_{i=1}^{n}\left(\frac{1}{3}|y_i(t)|^3+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}\int_{t-\tau_j}^{t}|y_j(s)|^3 ds\right), \tag{12}$$

where γ_j $(j=1,2,\dots,n)$ are any real number.

Calculating the upper right derivative D^+V of V along the solution of Eq. (5), we get

$$\begin{aligned}
D^+V &\leqslant \sum_{i=1}^{n}\left[|y_i(t)|^2\left(-c_i|y_i(t)|+\sum_{j=1}^{n}|a_{ij}|\mu_j|y_j(t)|+\sum_{j=1}^{n}|b_{ij}|\mu_j|y_j(t-\tau_j)|\right)+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}[|y_j(t)|^3-|y_j(t-\tau_j)|^3]\right]\\
&=\sum_{i=1}^{n}\left[-c_i|y_i(t)|^3+\sum_{j=1}^{n}|a_{ij}|\mu_j|y_i(t)|^2|y_j(t)|+\sum_{j=1}^{n}|b_{ij}|\mu_j|y_i(t)|^2|y_j(t-\tau_j)|\right.\\
&\quad\left.+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}[|y_j(t)|^3-|y_j(t-\tau_j)|^3]\right]\\
&=\sum_{i=1}^{n}\left[-c_i|y_i(t)|^3+\sum_{j=1}^{n}|a_{ij}|(\mu_j^{\alpha_j}|y_i(t)|)(\mu_j^{\beta_j}|y_i(t)|)(\mu_j^{\gamma_j}|y_j(t)|)\right.\\
&\quad\left.+\sum_{j=1}^{n}|b_{ij}|(\mu_j^{\alpha_j}|y_i(t)|)(\mu_j^{\beta_j}|y_i(t)|)(\mu_j^{\gamma_j}|y_j(t-\tau_j)|)+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}[|y_j(t)|^3-|y_j(t-\tau_j)|^3]\right],
\end{aligned} \tag{13}$$

where $\alpha_j, \beta_j, \gamma_j$ are any real numbers with $\alpha_j+\beta_j+\gamma_j=1$ $(j=1,2,\dots,n)$. Now estimating the right side of Eq. (13) by using of the inequality $3abc\leqslant a^3+b^3+c^3$ $(a,b,c\geqslant 0)$, we have

$$\begin{aligned}
D^+V &\leqslant \sum_{i=1}^{n}\left[-c_i|y_i(t)|^3+\sum_{j=1}^{n}|a_{ij}|\frac{1}{3}[(\mu_j^{\alpha_j}|y_i(t)|)^3+(\mu_j^{\beta_j}|y_i(t)|)^3+(\mu_j^{\gamma_j}|y_j(t)|)^3]+\sum_{j=1}^{n}|b_{ij}|\frac{1}{3}[(\mu_j^{\alpha_j}|y_i(t)|)^3+(\mu_j^{\beta_j}|y_i(t)|)^3\right.\\
&\quad\left.+(\mu_j^{\gamma_j}|y_j(t-\tau_j)|)^3]+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}[|y_j(t)|^3-|y_j(t-\tau_j)|^3]\right]\\
&=\sum_{i=1}^{n}\left[-c_i|y_i(t)|^3+\sum_{j=1}^{n}|a_{ij}|\frac{1}{3}[(\mu_j^{\alpha_j}|y_i(t)|)^3+(\mu_j^{\beta_j}|y_i(t)|)^3+(\mu_j^{\gamma_j}|y_j(t)|)^3]+\sum_{j=1}^{n}|b_{ij}|\frac{1}{3}[(\mu_j^{\alpha_j}|y_i(t)|)^3+(\mu_j^{\beta_j}|y_i(t)|)^3]\right.\\
&\quad\left.+\frac{1}{3}\sum_{j=1}^{n}|b_{ij}|\mu_j^{3\gamma_j}|y_j(t)|^3\right]\\
&=\sum_{i=1}^{n}\left[-c_i|y_i(t)|^3+\sum_{j=1}^{n}\frac{1}{3}|a_{ij}|(\mu_j^{3\alpha_j}+\mu_j^{3\beta_j})|y_i(t)|^3+\sum_{j=1}^{n}\frac{1}{3}|a_{ji}|\mu_i^{3\gamma_i}|y_i(t)|^3+\sum_{j=1}^{n}\frac{1}{3}|b_{ij}|(\mu_j^{3\alpha_j}+\mu_j^{3\beta_j})|y_i(t)|^3\right.\\
&\quad\left.+\sum_{j=1}^{n}\frac{1}{3}|b_{ji}|\mu_i^{3\gamma_i}|y_i(t)|^3\right]\\
&=\sum_{i=1}^{n}\left[-\left(c_i-\frac{1}{3}\sum_{j=1}^{n}[(|a_{ij}|\mu_j^{3\alpha_j}+|a_{ij}|\mu_j^{3\beta_j}+|a_{ji}|\mu_i^{3\gamma_i})+(|b_{ij}|\mu_j^{3\alpha_j}+|b_{ij}|\mu_j^{3\beta_j}+|b_{ji}|\mu_i^{3\gamma_i})]\right]|y_i(t)|^3\\
&\leqslant -r_1\sum_{i=1}^{n}|y_i(t)|^3,
\end{aligned} \tag{14}$$

where $r_1=\min_{1\leqslant i\leqslant n}[-(c_i-\frac{1}{3}\Sigma_{j=1}^{n}((|a_{ij}|\mu_j^{3\alpha_j}+|a_{ij}|\mu_j^{3\beta_j}+|a_{ji}|\mu_i^{3\gamma_i})+(|b_{ij}|\mu_j^{3\alpha_j}+|b_{ij}|\mu_j^{3\beta_j}+|b_{ji}|\mu_i^{3\gamma_i}))]>0$. A consequence of Eq. (14) is that

$$V(y)(t)+r_1\int_0^t\sum_{i=1}^{n}|y_i(s)|^3ds\leqslant V(y)(0). \tag{15}$$

It follows from Eq. (15) that

$$\int_0^{+\infty}\sum_{i=1}^{n}|y_i(t)|^3dt<+\infty. \tag{16}$$

Applying the methods similar to Theorem 1, we can prove that $|y_i(t)|,|y_i(t)|^3$ are uniformly continuous on $(0,+\infty)$, then it can known from Eq. (16) that

$$\lim_{t\to+\infty}\sum_{i=1}^{n}|y_i(t)|^3=0. \tag{17}$$

It follows from Eq. (17) that the equilibrium of the DCNN (1) is also globally asymptotically stable. This completes the proof.

Remark 2. By comparing the two theorems, it is easily found that Theorem 1 is equivalent to Theorem 2 as $\mu_j\equiv1$, $a_{ij}=a_{ji}$, $b_{ij}=b_{ji}$ $(i,j=1,2,\ldots,n)$. In addition, take $n=2$, $f_i(x)=\frac{1}{2}(|x+1|-|x-1|)$, $c_i=1$ $(i=1,2)$, clearly, the function f_i satisfy the hypotheses (H_1) and (H_2) above, and $\mu_i\equiv1$ $(i=1,2)$, at this time, we give the following two examples and can easily check that (i) as $n=2$, $\mu_i\equiv1$, $c_i=1$ $(i=1,2)$, $a_{11}=b_{11}=a_{22}=b_{22}=0.25$, $a_{12}=b_{12}=0.3$, $a_{21}=b_{21}=0.15$, DCNN satisfies the conditions of Theorem 1, but not those of Theorem 2; (ii) as $n=2$, $\mu_i\equiv1$, $c_i=1$ $(i=1,2)$, $a_{11}=b_{11}=0.25$, $a_{22}=b_{22}=0.15$, $a_{12}=b_{12}=0.2$, $a_{21}=b_{21}=0.3$, DCNN satisfies the conditions of Theorem 2, but not those of Theorem 1, i.e., conditions of Theorem 1 are independent of conditions of Theorem 2, in the sense that for any one of them there exists a network which satisfies one but not the other.

III. CONCLUSION

In this paper, we have derived two main theorems which did not assume the symmetry of the connection matrix $(a_{ij})_{n\times n},(b_{ij})_{n\times n}$, and only assumed the output of the cell (i.e., the nonlinear properties of the cell) with (H_1) and (H_2) above. Also, the theorens did not contain each other, and did not require differentiable or strictly monotonously increasing; for this reason, the sufficient conditions established in the two Theorems above have a wider adaptive range, and these conditions can be applied to design unconditional globally stable cellular neural networks with delays (DCNN), which possess highly important significance in some applied fields, for instance, the global optimization problem. In addition, the results in this paper are easily verifiable and the each of the outputs of the cell f_j $(j=1,2,\ldots,n)$ are quite general, and it has very wide adjustable leeway because our sufficient criteria possess many adjustable real parameters, which are of significance in the design of DCNN.

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